Porro, cum Angulum ſic, ut dictum eſt, definiverat, p. 67; ſubjungit, p. 68. *Quodſi magnitudines illæ ſint duæ lineæ, comprehenſus ab iis angulus, Planus vocabitur:* quaſi quidem de *Triangulis ſphæricis* nil unquam inaudiverit; nec alius eſſe poſſit ſuperficialis angulus, quam in *Plano.*

Adhæc, illud *duarum pluriumve*, de *Lineis* non tuto dicitur. *Trium* enim linearum concurſus, non angulum, ſed angulos ſaltem duos, conſtituunt; non enim lineæ plures duabus ad unum ſuperficialem angulum conſtituendum concurrunt. Item, cum p. 67. Angulum in genere per *duarum pluriumve*, &c. definiverat; Angulum p. 68. *una vel pluribus* ſuperficiebus comprehenſum ait (& unâ quidem angulum verticalem Coni comprehenſum;) quaſi quidem *una*, fuerit, *duæ* vel *plures.*

Inſuper, quid demum illud eſt, quod per *breviſſimam diſtantiam* inſinuatum vult? Quippe in ipſo concurſus puncto, *Nulla eſt diſtantia*; extra illud, *nulla minima*: nulla utique aſſignari poterit, qua non ſit minor: ſed revera tota hæc, quam de *Angulo* notionem concipit, eſt parum ſana. Definiendus utique eſt non per *diſtantiam* ſeu *remotionem*, ſed per *Inclinationem.* qnod ex *Euclidis* definitione didiciſſet.

Deniq; (ne multus nunc ſim) p. 171. in duabus his Quadraticarum æquationum formulis $aa - ca + dd = 0$, & $aa + ca + dd = 0$; utramque radicem *affirmativam* eſſe pronunciat. quod omnino ſecus eſt. Et quidem in priore, Radix utraque Affirmativa; ſed in poſteriore, Negativa utraque.

Atque hæc quadem, ex multis pauca, ſi non ſufficiant, ut ex ungue Leonem æſtimes, plura facile congerentur. Num autem hos *Incuria*, an *Inſcitia*, errores fuderit (prout ipſe *pag. ult.* diſtinguit) non determino. Vale.

Hæc Dn. *Walliſius* epiſtola una; cui poſtea ſubmiſit alteram, 18. *Julii* ad me ſcriptam, quam iſtoc menſe, ob alia, non licebat typis committere; nec quidem licet hoc ipſo: ne ſcil. haſce ſchedulas, publicationi variorum, idque imprimis ſermone *Anglico*, deſtinatas, diſceptationibus *Latinis* compleamus. Proxima occaſione, quæ idem *Author* porro notanda invenit vel in *unico primo* Capite *Synopſeos Laurentianæ*, Lectori (cum particularia flagitet Dn. Du Laurens) ob oculos ſiſtemus.

An Account of Two Books.

I. *R. de GRAAF Med. D. de VIRORUM ORGANIS GENERATIONI INSERVIENTIBUS, &c.* Ludg. Bat. 1668. *in* 12°.

This Treatiſe was promiſed by the Author in a printed *Epiſtle* of his, which we gave an account of in *April* laſt, *Num.* 34. p. 663. There being at the ſame time publiſht a *Prodromus* of *Joh. Van Horne*, ſuſpecting, that the Obſervations of *De Graef* were much the ſame with his upon this Subject; we do now upon the peruſal of this Book, find chiefly theſe conſiderable Differences between them.

Firſt, the ſaid *Van Horne* makes the *Spermatick Artery* in man to goe to the *Teſticles* in a *winding*, but *De Graaf*, in a *ſtreight* way.

Secondly, the *former* affirms, that the *vaſa deferentia* have no communication with the *veſiculæ ſeminales*; but the latter maintains, and demonſtrateth it to the Ey, there is ſo great a commerce betwixt them, *ut ſemen dum à Teſtibus per vaſa differentia affluens in Urethram effluere nequit, propter carunculam clauſam; neceſſariò influat in Veſiculas, in iiſque pro futuro coitu reſervetur.*

Thirdly, the *former* is of opinion, *triplicem eſſe materiam ſeminis*; but *De Graaf* will have but *one only*; anſwering the Arguments, uſed both by *Van Horne* and *Dr. Wharton* to prove that *triplicity*.

But that, which *De Graaf* much inſiſts on in this Book, is, to ſhew what is the *true Subſtance* of the *Teſticles*, and to vindicate the Diſcovery thereof to himſelf; affirming poſitively, that no man, before him ever knew the truth of it. * For the making out of which, he firſt denyeth, that the *Teſtes* are *glandulous*, or *pultaceous*; and then affirms that their ſubſtance is nothing elſe but a *Congeries minutiſſimorum vaſculorum ſemen conficientium, quæ ſi abſque ruptione diſſoluta ſibi invicem adnecteretur, facile viginti ulnarum longitudinem excederent.* Which he affirms, he can prove by ocular Demonſtration.

* See the Letter of Doctor *Tim. Clark*, N. 35. p. 681.

Then he ſheweth, how the ſeminal veſſels paſs *è Teſtibus ad Epididymides*, vid. not by *one* Trunck (as Dr. *Highmore* thinks) but by *6.* or 7. ſmall *ductus*'s; aſſigning the cauſe, why Doctor *Highmore* did not ſee them.

Further he examines, An ſemen in teſtibus conficiatur; utrum ex Sanguine vel ex Lympha? quomodo elaboretur, craſſeſcat, lacteſcat: qua via à Teſtibus ad Urethram excurrat.

Moreover he endeavours to prove, *Veſiculas ſeminales ordinatas eſſe non ſeminis generationi, ſed receptioni & aſſervationi.*

He alſo obſerveth concerning the *ſeminal matter*, that 'tis compoſed *ex duplici materia*, which after *Ariſtotle*, he calls λόγον σπερματικὸν καὶ ὄγκον σπερματικὸν, conſidering this twofold matter like *Dough* and *Ferment*, this infecting and quickning that, and the groſſer part being a conſervatory and vehicle to that, which is moſt elaborate.

When he examins the *Penis*, he taketh notice, *Omnes hactenus*

nus Anatomicos perperam aſsignaſſe uſum muſculorum Penis, quos Erectores *appellant; Eorum quippe provinciam non eſſe, Penem erigere, & dilatare Urethram, cum omnis Muſculi actio ſit contractio, quæ extenſioni contraria eſt; eos potius* Penem *verſus interiora retrahere quam erigere: Interim, hoſce Penis Muſculos, coarctando corpora nervoſa circa eorum exortum, materiam ſeminalem verſus Penis partem anteriorem propellere, atque hac ratione corporum nervoſorum diſtenſione erectionem augere.*

Before we conclude this Account, we cannot but take notice, that the Author occaſionally inſerts in this Book divers curious and remarkable Examples and Obſervations; ſome whereof are.

1. Concerning thoſe, that are born, either *abſque Teſtibus*; or, *cum Teſticulo uno*; or, *cum tribus, idque hæreditario per aliquot familias, admodum fæcundas.*

2. About the *ſitus præternaturalis Teſticulorum, generationis tamen virtutem non impedientis.*

3. Concerning *lacteſcent Bloud* in a man living at *Delft* in *Holland*, whoſe Bloud alwayes turn'd into Milk, when let out either by venæ-ſections, or by bleeding at the Noſe, or by a wound. V. pag. 84, 85. Compare *Numb. 6.* pag. 100, 117, 118. and *Numb.* 8. pag. 139. of theſe *Tranſactions.*

4. Concerning the ſtrange alteration made in Femals, *ab Aura ſeminali:* quod confirmat exemplo felis, diu ſugentis (idque ad integram fere ſui nutritionem) lac mammarum caniculæ, per aliquot annos à coitu prohibitæ; deinceps vero, poſtquam catella admiſerat canem, nunquam ab eo tempore lac ex mammis ejus exſugere volentis.

5. About a ſtrange *Hæmorrhagy* per *Penem*, which amounted to 14. pound, in a Porter of 52. years old, falling down with a heavy load upon a board, laid over a ditch, and ſo turning about, when the ſaid porter trod upon it, that it caſt him down upon its edge, turn'd between his legs; yet the Patient by the skill and care of our Author recover'd.

6. Various Obſervations of Clyſters and Suppoſitories, caſt up by Vomits, p. 195, 196.

7. Several wayes of performing unbloudy diſſections of Animals, p. 228, 229, &c.

II. LOGARITHMOTECHNIA NICOLAI MERCATORIS. *Concerning which we ſhall here deliver the account of the Judicious* Dr. I. Wallis, *given in a Letter to the Lord Viſ-count* Brouncker, *as follows*;

Incidebam heri (Illuſtriſſime Domine) in D. *Mercatoris Logarithmotechniam*, nuper editam. Quæ ita mihi placuit, ut non prius dimiſerim quàm perlegiſſem totam. Et quamquam pauca quædam, Phraſeologiam quod ſpectat ſeu loquendi formulas nonnullas, mutata mallem; ſunt tamen ipſa ſenſu ſuo ſana: Eaque quæ ſuperſtruitur Doctrina, Logarithmos expedite atque ſubtiliter conſtruendi, perſpicue ſatis atque ingenioſe traditur.

Quæ huic ſubjungitur *Quadratura Hyperbolæ*, elegans admodum eſt atque ingenioſa. Nempe ad hunc ſenſum. V.Fig.1.

Poſtquam in Hyperbola MBF, (cujus Aſymptotæ AN, AH, ad angulum rectum coeunt) oſtenderat, prop. 14. Rectangula BIA, FHA, spA, &c. (ductis BI, FH, sp, &c, parallelis Aſymptotæ AN,) invicem eſſe æqualia; adeoque latera habere reciproce proportionalia; (quæ nota eſt Hyperbolæ proprietas:) Poſitis $AI = BI = 1$, & $HI = a$: oſtendit, prop. 15. $FH = \frac{1}{1+a}$ (Nempe propter analogiam AH. AI :: BI. FH. hoc eſt. $1+a.\ 1 :: 1.\ \frac{1}{1+a}.$ Sed & (quod Dividendo 1, per $1+a$ oſtenditur,) $\frac{1}{1+a} = 1 - a + a^2 - a^3 + a^4$ &c.] (continuatis deinceps, ipſius *a* poteſtatibus, alternatim negatis & affirmatis.)

$$
\begin{array}{r}
1+a)1(1,-aa^2,+,a^3,+,a^4,\&c \\
\underline{1+a} \\
-a \\
\underline{-a\ \ -a^2} \\
+a^2 \\
\underline{+a^2\ \ +a^3} \\
-a^3 \\
\underline{-a^3\ \ a^4} \\
+a^4 \\
\&c.
\end{array}
$$

Cumque hoc perinde obtineat, ubicunque ultra punctum I, ponatur H. Poſitis, ut prius $AI = 1$; hujuſque continuatione qualibet, ut $Ir = A$; quæ intelligatur in æquales partes innumeras dividi, quarum quælibet, ut Ip. pq, &c. dicatur *a*; adeoque Ip, Iq, &c, ſint *a*, 2a, 3a, &c. uſque ad A: Quæ his reſpondent rectæ ps, qt, &c. uſque ad *ru*, (ſpatium BI ru complentes) ſunt,

$$1 - a + a^2 - a^3 + a^4 \text{ \&c.}$$
$$1 - 2a + 4a^2 - 8a^3 + 16a^4 \text{ \&c.}$$
$$1 - 3a + 9a^2 - 27a^3 + 81a^4 \text{ \&c.}$$

& ſic deinceps uſque ad

$$1 - A + A^2 - A^3 + A^4 \text{ \&c.}$$

Cum itaque ſint

$$\begin{array}{ll} 1 + 1 + 1 \text{ \&c. (uſque ad ultimum)} & = A \\ a + 2a + 3a \text{ \&c. (uſque ad } A) & = \tfrac{1}{2}A^2 \\ a^2 + 4a^2 + 9a^2 \text{ \&c. (uſque ad } A^2) & = \tfrac{1}{3}A^3 \\ a^3 + 8a^3 + 27a^3, \text{ \&c. (uſque ad } A^3) & = \tfrac{1}{4}A^4 \end{array}$$

& ſic deinceps:

(quod oſtendit ille prop. 16, eſtque à me alibi demonſtratum:) Recte colligit, prop. 17. Expoſitum ſpatium Hyperbolicum $BIru = A - \frac{1}{2}A^2 + \frac{1}{3}A^3 - \frac{1}{4}A^4 + \frac{1}{5}A^5$, &c. Adeoque ſi (aſſignato, ipſi $A = Ir$, valore ſuo in numeris, ut res poſtulaverit,) diſtribuantur in duas claſſes $A, \frac{1}{3}A^3, \frac{1}{5}A^5$, &c. (poteſtates affirmatæ,) & $\frac{1}{2}A^2, \frac{1}{4}A^4$, &c. (poteſtates negatæ;) harumque Aggregatum, ex Aggregato illarum, ſubducatur; Reſiduum erit ipſum $BIru$ ſpatium Hyperbolicum.

Nequis autem operam luſum iri exiſtimet, propter Addendorum ſeriem in utraque claſſe infinitam; adeoque non abſolvendam: Hinc incommodo medelam (tacitus) adhibet: ponendo $A = 0.1$, vel $A = 0,21$, aliive fractioni decimali æqualem, adeoque minorem quam 1: (Hoc eſt, ſumpta Ir minore quam $AI = 1$.) Quo fit, ut poſteriores ipſius A poteſtates tot gradibus infra Integrorum ſedem deſcendant, ut merito negligi poſſint.

Exempli gratia; poſitis $AI = 1$, & $Ir = 0\,21$. erit

$$\begin{array}{ll} A = 0,21 & \\ \tfrac{1}{3}A^3 = 0,003087 & \tfrac{1}{2}A^2 = 0,02205 \\ \tfrac{1}{5}A^5 = 0,000081682 & \tfrac{1}{4}A^4 = 0,000486202 \\ \tfrac{1}{7}A^7 = 0,000002572 & \tfrac{1}{6}A^6 = 0,000014294 \\ \tfrac{1}{9}A^9 = 0,000000088 & \tfrac{1}{8}A^8 = 0,000000472 \\ \tfrac{1}{11}A^{11} = 0,000000003 & \tfrac{1}{10}A^{10} = 0,000000016 \\ \hline + 0,213171345 - & 0,022550984 = 0,190620361 = BIru \end{array}$$

Quæ eſt brevis Synopſis Quadraturæ ſuæ ſatis elegans.

Diſſimulandum interim non eſt; ſiquis totius $BIHF$ ſpatii (cujus latus IH longius intelligatur quam AI) quadraturam poſtulet; rem non ita feliciter ſucceſſuram: propter medelam, quam modo diximus, malo minus ſufficientem. Cum enim jam ponenda ſit $A > 1$; manifeſtum eſt, poſteriores ipſius poteſtates, altius in Integrorum ſedes penetraturas, adeoque minime negligendas.

Huic autem incommodo, levi conſtructionis immutatione, facile ſubvenitur. Vid. Fig. 1.

Cæteris utique ut prius conſtructis; Quadrandum exponatur $HFur$ ſpatium

tium; (cujuſcunque fuerit longitudinis AH; puta major minorve quam AI, vel huic æqualis: ſumptoque ubivis inter A & H, puncto r; puta ultra citrave punctum I, vel in ipſo I puncto:) Ponantur autem (non, ut prius $AI = 1$, & $Ir = A$: ſed) $AH = 1$; & $Hr = A$, quæ intelligatur in æquales partes innumeras dividi, quarum quælibet ſit *a*. Erunt itaque, poſt $AH = 1$, reliquæ deinceps decreſcentes $1-a$, $1-2a$, $1-3a$, &c. uſque ad $Ar = 1-A$. Item, propter æqualia Rectangula FHA, urA, BIA, &c. puta, $= b^2$: Erit $HF = \frac{b^2}{1}$; reliquæque deinceps $\frac{b^2}{1-a}$, $\frac{b^2}{1-2a}$, $\frac{b^2}{1-3a}$, &c, uſque ad $ru = \frac{b^2}{1-A}$ ſpatium HFur complentes. (Quæ omnia oſtenſa ſunt, in mea *Arithmetica Infinitorum*, prop. 88, 94, 95.)

Factaque Diviſione; reperietur $\frac{b^2}{1-a} = b^2 + b^2a + b^2a^2 + b^2a^3 + b^2a^4$, &c. Hoc eſt, b^2 in $1 + a + a^3 + a^2 + a^4$, &c. (ſumptis ipſius *a* poteſtatibus continue ſequentibus affirmatis omnibus.) Cumque de reliquis idem ſit judicium; erunt rectæ omnes, ipſis HF & ru interjectæ,

$$1-a)\,b^2\,(b^2, + b^2a, + b^2a^2, + b^2a\ \&c.$$
$$\underline{b^2 - b^2a}$$
$$+ b^2a$$
$$\underline{+ b^2a - b^2a^2}$$
$$+ b^2a^2$$
$$\underline{+ b^2a^2 - b^2a_3}$$
$$+ b^2a_3$$
$$\underline{+ b^2a^3 - b^2a_4}$$
$$+ b^2a^4$$
&c.

$$\left.\begin{array}{l} 1 + a + a_2 + a^3 + a^4\ \&c. \\ 1 + 2a + 4a^2 + 8a^3 + 16a^4\ \&c. \\ 1 + 3a + 9a^2 + 27a_3 + 81a^4\ \&c. \\ \text{\& ſic deinceps uſque ad} \\ 1 + A + A_2 + A^3 + A^4\ \&c. \end{array}\right\}\ \text{in}\ b^2.$$

Omniūmq; Aggregatū, $A + \frac{1}{2}A^2 + \frac{1}{3}A_3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ &c, in $b^2 = FHru$. (per *Arithm. Infin.* prop. 64.)

Exempli gratia.
Poſitis $AH = 1$.
$Hr = A = 0,21$
$AI = b = 0,\ 1$
Adeoque $b^2 = 0,01$

Erunt $A = 0,21$
$\frac{1}{2}A^2 = 0,02205$
$\frac{1}{3}A_3 = 0,003087$
$\frac{1}{4}A^4 = 0,00048623-$
$\frac{1}{5}A^5 = 0,000081682+$
$\frac{1}{6}A^6 = 0,000014294+$
$\frac{1}{7}A^{7}_{8} = 0,000002573-$
$\frac{1}{8}A^9 = 0,000000473-$
$\frac{1}{9}A = 0,000000088+$
$\frac{1}{10}A^{10} = 0,0000000017-$
$\frac{1}{11}A^{11} = 0,000000003+$

Horum ſumma — 0,235722333
Ducta in $b^2 = 0,01$
Exhibet ——— $0,002357 2 2 3 3 3 = FHru$

Qua-

$$\text{Qualium } 1. = ANGN \begin{cases} \text{Quadrato,} \\ \text{Rhombo,} \end{cases} \text{ſi angulus A ſit} \begin{cases} \text{Rectus.} \\ \text{Obliquus.} \end{cases}$$

Quæ quidem tam abſoluta eſt tamque expedita Hyperbolæ quadratura, ut neſciam an melior ſperari debeat.

Atque hæc ſunt quæ hac de re ſcribenda duxi. Quæ ſi D. *Mercatori* impertiveris; non diſplicebit, credo, hæc ſuæ Quadraturæ facta acceſſio.

Poſſe hæc ad *Logarithmorum* inventionem accommodari, non eſt quod moneam: Sed & ad *Summam Logarithmorum* inveniendum: (quam inquirit ille prop. 19.) Nempe, Poſitis $AH = 1$, $AI = IB = b$, (ut prius) planoque $BIHF = pl$. Erit $pl - b^2 + b^3 = BIps + BIqt + BIru$, &c. uſque ad $BIHF$. Si autem non ab ipſa BI incipiatur; ſed ultra citrave, puta à *ps*: Poſita $pH = a$ & $psFH = pl$. erit (univerſaliter) $pstq + psur$ &c (uſque ad $psFH$) $= pl - ab^2$: qualium 1, æquetur cubo ipſius AH.) Quod alias, ſi opus erit, demonſtrabitur. Tu interim, Illuſtriſſime Domine, Vale.

Oxon. d. 8. *Julii*, 1668.

Fig 1.

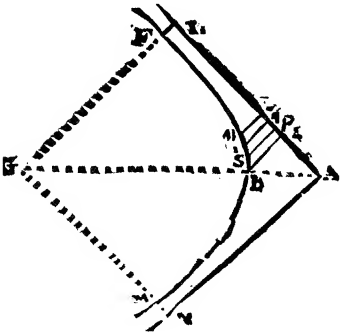
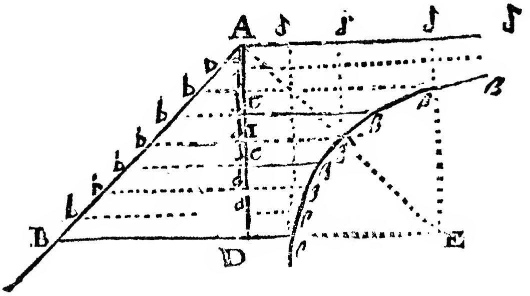


Fig 2.



The Demonſtration

Promiſed at the end of the foregoing Letter, follows in another from the ſame Author to the ſame Noble Lord, thus;

Petis (Illuſtriſſime Domine) per literas tuas *Aug*.3. datas (quas heſterna nocte accepi) ut demonſtrare velim methodum meam, Logarithmorum ſummam inveniendi, quam literis meis *Julii* 8. datis, breviſſime inſinuaveram.

Quæ quidem, cum ſit cum Ungularum doctrina (quam alibi trado) connexa; opus erit ut utramque ſimul exponam: ſed & rem totam (quam D. *Mer-*

catoris

vatoris figuræ & methodo quantum res ferebat accommodaveram) ad principia mea revocatam ab origine repetam. V. Fig. 2.

Oſtenſum eſt, in mea *Arithmetica Infinitorum*, prop. 95. Spatium Hyperbolium ADββδ (in infinitum continuatum à parte βδ, ſed à parte Dβ ubivis terminatum,) Figuram eſſe quam ex *Primanorum Reciprocis* conflatam appello, Prop. 88. definitam: Cujus nempe Ordinatim—applicatæ dβ, dβ, ſint Primanis (ſeu *A*rithmetice proportionalibus) db, db, (Triangulum complentibus) adeoque ipſis dA, dA, (ſuis à vertice diſtantiis) Reciproce Proportionales. Hoc eſt, (poſito AD = d; & rectangulo ADβ = b^2; particuliſque infinite exiguis a, a, &c;) ſi à vertice Aδ incipias $\frac{b^2}{o}$, $\frac{b^2}{a}$, $\frac{b^2}{2a}$, $\frac{b^2}{3a}$, &c. uſque ad $\frac{b^2}{d} = D\beta$: vel, ſi à baſe Dβ incipias, $\frac{b^2}{d}$, $\frac{b^2}{d-a}$, $\frac{b^2}{d-2a}$, $\frac{b^2}{d-3a}$, &c. uſque ad $\frac{b^2}{d-d} = A\delta$ infinitæ, (nempe, ſi ad Verticem uſque proceſſum continuaveris;) vel, uſque ad $\frac{b^2}{d-A} = C\beta$, (poſito DC = A,) ſi continuaveriis uſque ad Cβ, ubivis intra Aδ & Dβ ſumptam. (Adeoque omnium Aggregatum, $\frac{b^2}{d} + \frac{b^2}{d-a} + \frac{b^2}{d-2a} + \frac{b^2}{d-3a}$, &c, eſt ipſum DCββ planum.)

Manifeſtum itaque eſt, (& ibidem prop. 94. oſtenſum) ſi intelligantur ſingulæ dβ, in ſuas à vertice diſtantias *Ad*, ductæ; hoc eſt, $\frac{B^2}{a}$ in *a*, $\frac{B^2}{2a}$ in 2a, (& ſic de reliquis;) erunt omnia rectangula Adβ; hoc eſt, rectarum dβ momenta reſpectu Aδ, (intellige, facta ex magnitudine in diſtantiam ductâ;) ſeu plana ſemiquadrantalem Ungulam (cujus acies Aδ) complentia, (eiſdem dβ rectis perpendiculariter inſiſtentia;) invicem æqualia. Quippe ſingula = b^2. (Quorum cum unum ſit AIVδ quadratum, erit AI = b.)

Adeoque Totius ADββδ (plani infiniti) ſeu omnium dβ illud complentium, momentum reſpectu rectæ Aδ, (ut axis æquilibrii;) ſeu Ungula ſemiquadrantalis eidem ADββδ inſiſtens (aciem habens Aδ;) ſunt totidem b^2; hoc eſt, db^2. (Ungula magnitudinis finitæ plano infinitæ magnitudinis inſiſtens.) Ejuſque pars plano ACββδ inſiſtens (propter AC = d — A.) ſimiliter oſtendetur æqualis ipſi d — A in b^2. ductæ; hoc eſt, $db^2 - Ab^2$. Adeoque pars reliqua, ipſi DCββ inſiſtens, æqualis ipſi Ab^2. Quod itaque eſt ejuſdem DCββ momentum reſpectu Aδ.

Atque hoc momentum per plani $DC\beta\beta$ magnitudinem, puta per *pl*, diviſum; exhibet plani diſtantiam Centri gravitatis ab $A\delta$, $\frac{ab^2}{pl}$: adeoque diſtantiam ejuſdem a $D\beta$, $d - \frac{ab^2}{pl}$.

Hæc itaque à $D\beta$ diſtantia, in *pl* (plani magnitudinem) ducta; exhibet $d\,pl - Ab^2$ ejuſdem $DC\beta\beta$ momentum reſpectu $D\beta$; ſeu Ungulam eidem $DC\beta\beta$ inſiſtentem, cujus acies ſit $D\beta$.

Hæc denique Ungula (cujus altitudo, in $D\beta$, nulla ſit, ſed, in $C\beta$, ipſi DC æqualis:) ſi ex planis ipſi $DC\beta\beta$ parallelis conflari intelligatur; erunt ea, $CD\beta\beta$, $Cd\beta\beta$, & ſic deinceps; hoc eſt, aggregatum omnium $Cd\beta\beta$, $Cd\beta\beta$, uſque ad $CD\beta\beta$.

Sunt autem ea plura (ut ex *Gregorii de Sancto Vincentio*, aliorumque poſt illum, doctrina conſtat) tanquam Logarithmi Arithmetice proportionalium Cd, Cd, &c. uſque ad CD; (ſeu a, $2a$, $3a$, &c. uſque ad A. Ergo Ungula ipſa, eſt eorundem Aggregatum. Hoc eſt (poſito $D = 1$,) $d\,pl - Ab^2 = pl - Ab^2$. Quod oſtendendum erat.

Porro; cum ſit $\frac{b^2}{d-a}$ $(= d\beta) = \frac{b^2}{d} + \frac{ab^2}{d2} + \frac{a^2b^2}{d3} + \frac{a^3b^2}{d4}$ &c

(Quod dividendo b^2 per $d-a$, patebit:) vel, poſito $d = 1$, (quó ipſius *d* poteſtates omnes deleantur,) $b^2 + ab^2 + a^2b^2 + a^3b^2$ &c. ſeu $1 + a + a^2 + a^3$, &c. in b^2. & ſimiliter $\frac{b^2}{d-2a} = \frac{b^2}{d} + \frac{2a\,b^2}{d^2} + \frac{4a^2\,b^2}{d^3} + \frac{8a^3\,b^2}{d^4}$ &c. $= b^2 + 2ab^2 + 4a^2b^2 + 8a^3b^2$ &c. $= b^2$ in $1 + 2a + 4a^2 + 8a^3$, &c. & ſimiliter in reliquis:

Erunt omnes $d\delta$, (ſpatium $DC\beta\beta$ complectentes,)

$$\left.\begin{array}{l} 1+a+a^2+a^3+a^4\ \&c. \\ 1+2a+4a^2+8a^3+16a^4\ \&c. \\ 1+3a+9a^2+27a^3+81a^4\ \&c. \\ \quad\text{\& ſic deinceps uſque ad} \\ 1+A+A^2+A^3+A^4\ \&c. \end{array}\right\}\ \text{in } b^2.$$

Adeoq; (per Arithm. Infin. prop.64.) omnium Aggregatum, ſeu ipſum $DC\beta\beta$ ſpatium, erit ——

$$A+\tfrac{1}{2}A^2+\tfrac{1}{3}A^3+\tfrac{1}{4}A^4+\tfrac{1}{5}A^5\ \&c,\ \text{in } b^2 = pl.$$

Qualium $1 = ABE\delta$ Quadrato vel Rhombo

Ideoque, Plani $DC\beta\beta$ momentum reſpectu $D\beta$; ſeu ſemiquadrantalis Ungula eidem inſiſtens cujus acies ſit $D\beta$; ſeu planorum aggregatum ipſam conſtituentium; ſeu Logarithmorum ſumma quos ea repreſentant, $d\,pl - Ab^2 = pl - Ab^2, = \frac{1}{2}A^2 + \frac{1}{3}A^3 + \frac{1}{4}A^4 + \frac{1}{5}A^5$ in b^2:

Qualium

Qualium Cubus (ſeu Rhombus ſolidus) A D E δ ſit 1.

Si vero non ponatur d = 1, ſed cujuſcunque magnitudinis: erit ſaltem

$$\frac{A}{d} + \frac{A^2}{2d^2} + \frac{A^3}{3d^3} + \frac{A^4}{4d^4} \text{ \&c. in } b^2. = pl.$$

Vel (poſito $\frac{A}{d} = e$) erit $e + \frac{1}{2}e^2 + \frac{1}{3}e^3 + \frac{1}{4}e^4$ &c. in $b^2 = pl.$ Qualium d^2 = A D E δ Quadrato vel Rhombo.

Ungulaque (ut prius) $d\,pl - A\,b^2$. Qualium d^3 = A D E δ Cubo, vel (ſi angulus A ſit obliquus) Rhombo ſolido.

Cumque A (poſito d = 1) vel *e* (quicunque ponatur valor ipſius *d*) ſit minor quam 1, (propter $A < d$:) illius poteſtates poſteriores ita continue decreſcunt, ut tandem negligi poſſint; planique valor *pl.* exhibeatur quantumlibet vero propinquus.

Atque hæc eſt, Illuſtriſſime Domine, Methodi, quam innuebam, ex meis principiis deductio, & demonſtratio brevis. Vale. *Oxon.* d. 5. *Aug.* 1668.

Some Illuſtration
Of the Logarithmotechnia *of M.* Mercator, *who communicated it to the Publiſher, as follows.*

Si quorum in manus incidit Logarithmotechnia mea, non inviti, opinor, adſpicient paucula hæc exempla, miram iſtius methodi facilitatem cum ſumma præciſione conjunctam oſtendentia.

Exponentes	Unitatis ordo		Binarii ordo	
1	1	$\dot{1}$	2	$\dot{2}$
2	1	$\dot{0}$,5	4	$\dot{2}$
3	1	$\dot{0}$,333333	8	$\dot{2}$,666666
4	1	0,25	16	4
5	1	$\dot{0}$,2	32	$\dot{6}$,4
6	1	0,166666	64	1$\dot{0}$,666666
7	1	$\dot{0}$,142857	128	1$\dot{8}$,285714
8	1	0,125	256	3$\dot{2}$
9	1	$\dot{0}$,111111	512	56,888888
10	1	0,1	1024	102,4

Duo ſunt ordines tabellæ, prior unitatis, alter binarii propago, quorum uterque denorum numerorum primorum Log-os producit, præter compoſitorum Log-os, qui & ipſi requiruntur.

Ex primo ordine

i

$\dot{0}5$

$0\dot{3}3333333$

$\dot{0}25$

$\dot{0}2$

$\dot{0}16666$

$\dot{0}1428$

$\dot{0}125$

$\dot{0}11$

$\dot{0}1$

$+10033534772$

$-\ 502516792$

$10536051564\frac{9}{10}$

$9531017980\frac{10}{11}$

i

$.\dot{0}5$

$.\dot{0}333333$

$\dot{0}25$

$\dot{0}2$

$+10000333353$

$-\ 500025$

$10050335853\frac{99}{100}$

$9950330853\frac{100}{101}$

Parimodo ex eodem ordine procedunt rationes $\frac{999}{1000}$, $\frac{1000}{1001}$, $\frac{9999}{10000}$, $\frac{10000}{10001}$, $\frac{99999}{100000}$, $\frac{100000}{100001}$.

Ex ſecundo ordine.

$\dot{2}$

$\dot{2}$

$\dot{2}66666666$

$\dot{4}$

$\dot{6}4$

1066666

$1\dot{8}2857$

$\dot{3}2$

$5\dot{6}89$

$1\dot{0}24$

$18\dot{6}$

$34\dot{1}$

$63\dot{0}$

$+20273255404$

$-\ 2041099724$

$\dot{2}$

$.\dot{2}$

$.\dot{2}666666$

$\dot{4}$

$.\dot{6}4$

$1\dot{0}$

$+\ 20002667306$

$-\ 200040010$

$20202707316\frac{98}{100}$

$19802627296\frac{100}{102}$

Haud ſecus ex eodem ordine eliciuntur rationes $\frac{998}{1000}$, $\frac{1000}{1002}$, $\frac{9998}{10000}$, $\frac{10000}{10002}$, $\frac{99998}{100000}$, $\frac{100000}{100002}$, &c.

1		$22314355128\frac{8}{10}$
2		$18232155680\frac{10}{12}$
3	$\dot{i} + \dot{2}$	$40546510808\frac{8}{12} = \frac{2}{3}$
4	ex pc. pag.	$10536051564\frac{9}{10}$
5	$\dot{2} + \dot{4}$	$28768207244\frac{9}{12} = \frac{3}{4}$
6	$\dot{3} + \dot{5}$	$69314718052\frac{2}{4} = \frac{1}{2} = L\ 2^{tii}$
7	$\dot{6} \times 3$	$207944154156\frac{1}{8} = L\ 8^{tii}$
8	$\dot{i} + \dot{7}$	$230258509284\frac{1}{10} = L\ 10^{tii}$
9	ex pc. pag.	$9531017980\frac{10}{11}$
10	$\dot{8} + \dot{9}$	$239789527264\frac{1}{11} = L\ 11^{tii}$
11	$\dot{3} + \dot{6}$	$109861228860\frac{1}{2} + \frac{2}{3} = L\ 3^{tii}$

Simi-

Similes ordines à 3^{rio}, 4^{rio}, & quovis alio numero derivari poſſunt, ſuas quiſque rationes exhibiturus.

Acquiſito Log-o 10^{rii}, conficienda eſt ſtatim tabella reducendorum Log-orum naturalium ad Tabulares, ut quævis ratio, ſimul ac inventa eſt, reducatur ad menſuram tabularium; ita enim Log-i compoſitorum, quorum ope ad primorum Log-os deſcenditur, ſimul fient Tabulares abſque reductione.

Fiat igitur, ut Log-us 10^{rii} non-tabularis 2302585, ad tabularem 10000000; ita 1, ad 4,3429448. Hic numerus bis, ter, quater & pluries ſumptus conſtituit tabellam reducendorum Log-orum naturalium ad tabulares, quam hic ſubjectam vides.

1	043429448190
2	086858896380
3	130288344570
4	173717792761
5	217147240951
6	260576689141
7	304006137332
8	347435585522
9	390865033712

Hujus igitur ope tabellæ, rationis $\frac{98}{100}$ menſura naturalis 20202707316 reducitur ad tabularem hoc modo:

2	086858896381
0	0
2	0868588964
0	0
2	08685890
7	3040061
0	0
7	30401
3	1303
1	043
6	26
	87739243069

Tum à Log-o 100^{rii} 20000000000000
auferatur rationis $\frac{98}{100}$ menſura 87739243069
reſtat 19912260756031 = L 98
unde ablato Log-2^{ii} 3010299956640
reſtat ——— 16901960800291 = L 99
cujus ſemis ——— 8450980400145 = L 7

Item rationis $\frac{100}{102}$ menſura naturalis 19802627296 reducta, fit 86001717619.

Ergo à Log-o 100^{rii} 20000000000000
adde rationis $\frac{100}{102}$ menſuram 86001717619
fit ——— 20086001717619 = L 102
unde ablato Log-o 6^{rii} 7781512503836
reſtat ——— 12304489213783 = L 17

Hic tabula numerorum primorum egregium uſum præſtare poteſt.

Sed & ejuſdem primi 17 Log-um abſque ambage invenire datur, dicendo: 20. 17 :: 10. 8|5; tum differentiæ inter 10 & 8|5 (nimirum 1|5) ſumendo quadrati ſemiſſem, cubi trientem, &c. tractandoque iſtum ordinem, ut ſuprà, inveniemus ſimul Log-os abſolutorum 23, 197, 203, 1997, 2003, &c.

1	1,5	$\dot{1}$,5
2	2,25	$\dot{1}$,125
3	3,375	$\dot{1}$,125
4	5,0625	$\dot{1}$,265625
5	7,59375	$\dot{1}$,51875
6	11,390625	$\dot{1}$,8984375

$$
\begin{array}{r}
\dot{1}5 \\
\dot{1}125 \\
\dot{1}125 \\
\dot{1}265625 \\
\dot{1}518 \\
\dot{1}89 \\
\dot{2}2 \\
\hline
+15114040 \\
-\ 1137845 \\
\hline
16251885\,\frac{17}{20} \\
13976195\,\frac{20}{23}
\end{array}
$$

Cæterùm iſthæc omnia, & longè plura ex prop. 13, 15, & 16 Logarithmotechniæ noſtræ apertè derivantur, non magis conſiderando hyperbolam, quàm ſi ea nuſquam in rerum naturâ extitiſſet. Quare fruſtra ſunt, qui hyperbolam ad conſtructionem Logarithmorum vel hilum conferre autumant; imo Logarithmorum ope quadrare hyperbolam, verius eſt. Id quod exemplo oſtendere haud pigebit. In diagrammate (Fig. 1.) ſit AH 74305816 partium, qualium AI eſt 1, & oporteat invenire aream BIHF.

Opus eſt ad eam rem tabella ſubjecta, quæ continet Log-os naturales ſuprà acquiſitos, in priori columna ab 1 uſque ad 9, in altera à 10 uſque ad 1000000000.

1	00000000000	02,30258509299
2	69314718052	04,60517018599
3	109861228860	06,90775527898
4	138629436104	09,21034037198
5	160943791232	11,51292546497
6	179175946912	13,81551055796
7	194591014904	16,11809565096
8	207944154156	18,42068074395
9	219722457720	20,72326583695

Tum prima figura numeri dati ſemper diſtinguatur à ſequentibus ſeparatrice, hoc modo: 7,4305816, & ipſi primæ figuræ ſemper adjiciatur 1, ita conflantur, hoc loco, 8. Quærenda eſt nunc rationis 8 ad 7,4305816 menſura naturalis. Id ut fiat commodius, dic: ut 8 ad 7,4305816; ita 1 ad 0,9288227, hunc quartum proportionalem aufer ab 1, reliquum 0,0711773 voco poteſtatem primam, quæ ducenda eſt in ſe ita, ut in facto idem numerus partium extet, qui erat in ipſo 0,0711773; productum 0,0050662 eſt poteſtas ſecunda, quæ rurſus ducatur in primam 0,0711773, ut idem numerus partium exſtet, prodit 0,0003606, quæ eſt tertia poteſtas; & eodem modo invenitur quarta 0,0000256, & quinta 0,0000018. Deinde

$$\left.\begin{array}{lr} \text{Poteſtas prima} & 0,0711773 \\ \text{Et ſecundæ ſemis} & 25331 \\ \text{Et tertiæ triens} & 1202 \\ \text{Et quartæ quadrans} & 64 \\ \text{Et quintæ pars quinta} & 4 \end{array}\right\}\text{addantur}$$

ſumma ——————— 0,0738374 eſt menſura rationis 8 ad 7,43058 16, eadem ſcilicet cum ratione 80000000 ad 74305816. Porro Log-us abſoluti 80000000 facile acquiritur ex ſuperiori tabella; cùm enim index primæ figuræ numeri 80000000 ſit 7, è regione 7^{rii} ex ſecunda columna excerpo Log-um abſoluti 10000000 (hoc eſt unitatis ſeptem cyphris affectæ)

$$\left.\begin{array}{lr} \text{qui reperitur} & 16,11809565 \\ \text{cui ſubſcribo Log-um } 8^{rii} & 2,07944154 \end{array}\right\}\text{addo}$$

ſumma eſt Log-us abſoluti 80000000 = 18, 19753719

ablata menſura rationis 80000000 ad 74300 5816 = 0, 0738374

reſtat Log-us abſoluti 74305816 = 18, 1236997, atque tanta eſt area B I H F.

Mantiſſæ loco accipe modum facillimum quadrandi quamvis hyperbolæ partem per Log-os tabulares. Dati numeri 74305816 Log-us tabularis eſt 7,87102278, per ſuperioris tabellæ columnam ſecundam reducendus ad naturalem, proditque eadem, quæ ſupra, area B I H F = 18, 123699872.

Poſtremo, ne quis hæſitationi locus reſtet, accipe, quo pacto ex Prop. 13, 15, 16. Logarithmot. calculum ſuperiorem derivem.

Differentia terminorum rationem quamvis exprimentium ſi concipiatur diviſa in partes æquales innumeras; compoſita erit ratio tota extremorum terminorum ex innumeris ratiunculis terminorum à minimo ad maximum infinitiſſima parte ipſius differentiæ ſe mutuo excedentium. Sin iidem illi termini innumeri accipiantur pro mediis Arithmeticis aliorum terminorum ſimili parte infinitiſſima diſtantium; ſumma omnium ratiuncularum poſterioribus hiſce terminis intercedentium deficiet à tota ratione extremorum, non niſi ſemiſſe primæ & ultimæ ratiuncularum à prioribus terminis contentarum, id eſt, ratiuncula minori, quam quæ ullis numeris exprimi poſſit. Quare poſito Maximo termino = 1, & parte infinitiſſima differentiæ = i, & menſura rationis minimæ itidem i; erit ut medium Arithmeticum terminorum rationis minimam proxime præcedentis, ad medium Arithmeticum terminorum ipſius minimæ; ita menſura minimæ, ad menſuram proxime majoris; hoc eſt:

$$\left.\begin{array}{lllll} 1-i\,.\,1::i\,.\,i+ii+i^3+ & i^4 & \&c. & \text{menſuræ ultimæ} \\ 1-2i\,.\,1::i\,.\,i+2ii+4i^3+ & 8i^4 & \&c. & \text{penultimæ} \\ 1-3i\,.\,1::i\,.\,i+3ii+9i^3+ & 27i^4 & \&c. & \text{antepenultimæ} \end{array}\right\}\text{add.}$$

ſit ſumma ratiuncul. $=3i+6ii+14i^3+36i^4$ &c. = numero terminorum,

rum, plus ſumma eorundem terminorum, plus ſumma quadratorum ab iiſdem, &c.

Sin minimus terminus ponatur $= 1$, manentibus cæteris ut ſupra; evadit ſumma ratiuncularum $= 3i - 6ii + 14i^3 - 36i^4$, &c.

Hinc data differentia terminorum $= 0|\underline{1}$, erit numerus terminorum $= 0|\underline{1}$, & per 16 Logarithmot. ſumma eorundem terminorum $= 0, 005$, & ſumma quadratorum $= 0, 000333$. At data differentia terminorum $= 0|\underline{10}$; numerus terminorum eſt $= 0,01$, & ſumma eorundem $= 0,00005$, & ſumma quadratorum $= 0,00000333$, &c.

Nota. Prop. IV. Logarithmot. Signa ſpeciebus intercedentia debebant eſſe alternatim affirmata & negata: atque ubicunque, Lector offenderit *infinitiſſimam*, legat *infiniteſimam*.

Errata.

Page 742. *l.* 25. put a *comma* after *open'd*, (which is material for the ſenſe.) p. 749. *l.* 16. r. *idque*. ibid. *l* 40. r. *magnitudinum*. p. 753. *l.* 20 r. $-a + a^2, -a^3$, p. 754. *l.* 19. r. *Huic*. p. 755. *l.* 11. r. $b^2 a^2 + b^2 a^3 + b^2 a^4$. ibid. *l.* 14. r $+ a^2 + a^3$. p. 756. in Fig. 1. the letters appearing obſcure, thoſe, that denote the ſmall lines parallel to the Aſymptote N A, are I B. *ps. qt. rx*. And the other capital letters are G F H. G B A. G M N.

In the *SAVOY*,

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